AML610 Homework 2 solutions

See hwk2\_solutions.R

Question #1



\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*

\*\*\*\*\*\* n = 1000

\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*

The second Poisson and sample moments are: 110 108.828

The fourth Poisson and sample moments are: 16710 16116.98

The mean, and variance of the mean are: 9.968 0.009476452

The variance, and variance of the variance are: 9.476452 0.1887562

The probability that the mean is drawn from mu= 10 is 0.7423668

We accept the null hypothesis that the mean is drawn from a Normal distribution with mean 10

The probability that the variance is drawn from var= 10 is 0.2281836

We accept the null hypothesis that the variance is drawn from a Normal distribution with mean 10

The probability that the mean is drawn from mu= 10.05 is 0.3995938

We accept the null hypothesis that the mean is drawn from a Normal distribution with mean 10.05

The probability that the variance is drawn from var= 9.95 is 0.2757281

We accept the null hypothesis that the variance is drawn from a Normal distribution with mean 9.95

\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*

\*\*\*\*\*\* n = 1e+05

\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*

The second Poisson and sample moments are: 110 109.7633

The fourth Poisson and sample moments are: 16710 16618.73

The mean, and variance of the mean are: 9.99106 9.9421e-05

The variance, and variance of the variance are: 9.9421 0.002071466

The probability that the mean is drawn from mu= 10 is 0.3699327

We accept the null hypothesis that the mean is drawn from a Normal distribution with mean 10

The probability that the variance is drawn from var= 10 is 0.2033146

We accept the null hypothesis that the variance is drawn from a Normal distribution with mean 10

The probability that the mean is drawn from mu= 10.05 is 3.397528e-09

We reject the null hypothesis that the mean is drawn from a Normal distribution with mean 10.05

The probability that the variance is drawn from var= 9.95 is 0.8621905

We accept the null hypothesis that the variance is drawn from a Normal distribution with mean 9.95

Question #2

As lambda gets larger, the Poisson distribution approaches the Normal distribution



Question #3

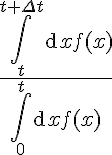
Prove that the Ehrlang distribution with k=2 is not memoryless

(NB: latex courtesy of http://www.texify.com/links.php)

The Ehrlang distribution with shape=2 and scale=theta is

http://www.texify.com/img/%5CLARGE%5C%21%5Cbegin%7Beqnarray%7Df%28x%7Ck%3D2%2C%5Ctheta%29%20%3D%20%7B%7B1%7D%5Cover%7B%5Ctheta%5E2%7D%7D%20x%20e%5E%7B-x/%5Ctheta%7D%5Cend%7Beqnarray%7D.gif

A distribution is memoryless iff

(Equation 1)

is independent of t.

We have

http://www.texify.com/img/%5CLARGE%5C%21%5Cbegin%7Beqnarray%7D%5Cint%20%7B%5Crm%20d%7Dx%20f%28x%29%20%3D%20%5Cint%20%7B%5Crm%20d%7Dx%20%20%7B%7B1%7D%5Cover%7B%5Ctheta%5E2%7D%7D%20x%20e%5E%7B-x/%5Ctheta%7D%5Cend%7Beqnarray%7D.gif

Without loss of generality, assume y=x/theta (which means dx=dy\*theta). This yields

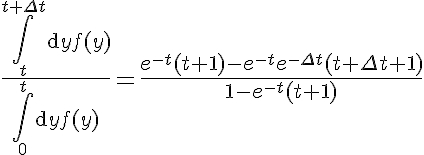
http://www.texify.com/img/%5CLARGE%5C%21%5Cbegin%7Beqnarray%7D%5Cint%20%20y%20e%5E%7B-y%7D%20%7B%5Crm%20d%7Dy%5Cend%7Beqnarray%7D.gif

(as an aside, can you see why theta is called a “scale” factor?)

This yields

http://www.texify.com/img/%5CLARGE%5C%21%5Cbegin%7Beqnarray%7D%5Cint%20%20y%20e%5E%7B-y%7D%20%7B%5Crm%20d%7Dy%20%3D%20-e%5E%7B-y%7D%28y%2B1%29%5Cend%7Beqnarray%7D.gif

In combination with Equation 1, this yields



There is no way to cancel out the terms involving t in the numerator and denominator. Therefore the Ehrlang distribution with k=2 is not memoryless.

The advantage of memoryless distributions in applications like individual based stochastic modeling is that you do not have to keep track of how long an individual has been in a state in order to calculate its probability of leaving that state (because the probability of leaving at any time is constant at any given time if the probability distribution is memoryless). This saves both memory and a lot of computation time, particularly if the population is large.