## Airline boarding schemes and optimization of passenger comfort

S.Towers ${ }^{1, *}$,
${ }^{1}$ Simon A. Levin Mathematical and Computational Modeling Sciences Center, Arizona State University, Tempa, AZ, USA

* E-mail: smtowers@asu.edu


## Abstract

## Objective

Using agent based models, we explore existing and novel airplane boarding paradigms to determine which schemes optimize customer comfort within the constraints of current average seat size, and the distribution of girth sizes of modern-day passengers.

## Background

While traditional airlines pre-assign seating, some lower-cost airlines facilitate speed of boarding using a "free-choice" boarding scheme, in which the passengers are assigned a boarding position. Once on the plane each passenger then picks the seat of their choice (for instance, choosing the seat that appears to afford them the most room). We explore if the latter paradigm actually results in more space per passenger.

## Methods

Using agent-based models, we simulate passengers of varying seated girth widths boarding planes, and examine the space available to each passenger, after accounting for their girth, and the girth of the passenger(s) next to them. We also present a novel simple boarding scheme designed to optimize available space to all passengers, whereby passengers board in order of size, from largest to smallest.

## Results

For free-choice boarding on a plane with a $3+3$ seat configuration (most commercial airplanes), we find that passengers $2 / 3$ of the way down the boarding line usually obtain the most spacious seating arrangement, even compared to all the people who boarded the plane first. Overall the average amount of space per passenger is identical on free-seating airlines compared to pre-assigned seating airlines.

The novel boarding paradigm where passengers board according to size helps to equalize the space available to all passengers, regardless of girth.

## Keywords

Agent based modeling; Aircraft; Transportation; Optimization;

## Introduction

In order to remain competitive in an increasingly cut-throat marketplace, airlines in recent years have focussed on transporting as many passengers as possible (Nadadur and Parkinson, 2009), as quickly as possible (Van Landeghem and Beuselinck, 2002; Steffen and Hotchkiss, 2012; Ferrari and Nagel, 2005; Bachmat et al., 2009; Tang et al., 2012), and for the greatest profit (Wollmer, 1992). These requirements have largely been met by ensuring that most planes fly full, and also by maximizing the number of seats on each plane by placing seats closer together front to back, and reducing seat widths; average airline seat widths have shrunk from a high of 18.5 " in the 1990 's, to an average of 17 " today (Ostrower and Michaels, 2013), despite the rising prevalence of obesity over that time period (Small and Harris, 2012), and the fact that seating room and comfort is a primary issue brought up by passengers on flight satisfaction surveys (Richards et al., 1978; TripAdvisor, 2013).

In 2001, Quigley et al published detailed anthropometric data obtained from measurements taken of people of various nationalities (Quigley et al., 2001). Based on the seated hip-width data collected by that study (which we will refer to as "girth" in this paper), we estimate that around $18 \%$ of Americans have girths that exceed the width of a 17 " airline seat plus the 2 " armrest width. While most airlines have policies that require that customers who cannot comfortably fit between the armrests pay for an additional seat, such policies are not consistently enforced, likely in large part due to social norms of etiquette that discourage public confrontation with such passengers (O'Neill et al., 2004; Harris and Small, 2009). Asking that the customer either deplane or pay for an additional seat after they have actually boarded the plane and the problem to other passengers has become apparent also risks flight delays, which are an unwelcome expense to airlines and an inconvenience to passengers.

Seats that are too narrow can have serious health implications, since prolonged immobility can cause deep venous thrombosis and edema (Arfvidsson et al., 2000; Brundrett, 2001). Indeed, studies have shown that $34 \%$ of airline passengers report numbness in their legs during a flight, and the risk is greater for larger passengers in economy size seats (Brundrett, 2001). Given this, and with such a significant fraction of Americans unable to fit comfortably into airline seats, it is somewhat surprising how little academic attention has been given to development of paradigms to relieve the problem within the existing constraints of current aircraft configurations. Indeed, in all the literature searches associated with this study, we found many articles related to optimization of airline operations, efficiency, and profit, but
found nothing about optimization of passenger comfort within existing seating plan paradigms.
In this analysis, we study whether or not the space available to seated passengers is optimized if they can choose which seat they take, compared to the average amount of space they would obtain with pre-assigned seating. Traditional airlines use pre-assigned seating, whereas some discount airlines, such as Southwest, have implemented a scheme whereby passengers are instead assigned a boarding position, and once on the plane can choose their own seat. In free-choice seating schemes, early boarding positions appear to be generally perceived as more favorable than later boarding positions, with passengers checking in as early as possible to obtain positions near the front of the line. Indeed, such positions are so desirable, that in January, 2013, Southwest began offering a boarding position upgrade option, whereby passengers could pay $\$ 40$ to be among the first 15 passengers to board the plane.

Using agent-based simulation studies, we examined the boarding schemes of assigned-seating and free-choice seating airlines, taking into account the seated girth width of a passenger, and the girths of the passenger(s) sitting next to them. In the model for the free-choice boarding, the passengers are given a boarding order and board the plane one at a time (Nyquist and McFadden, 2008), and then selfishly seek out the seat that maximizes the amount of room available to them (i.e. the seat space remaining after their girth and their neighbor's girth have been taken into account). With the simulations, we estimated the probability distributions of the spare seating room available to the passengers for each of the boarding schemes. We also examined the conditions under which an upgrade to the front of the line is advantageous in free-choice boarding.

As we will discuss, we find that the free-choice boarding scheme is no better than that of a preassigned seating paradigm in the marginal probability distributions for spare seating room available to the customers, despite the fact that each passenger can chose the seat that best optimizes their own personal comfort. However, the free-choice boarding paradigm does produce sharp disparities in the available spare seating room for some of the passengers on the plane, and we find some surprising results; on a plane with a $3+3$ seat configuration, passengers who board $2 / 3$ of the way down the line will on average end up with almost twice the available spare seating room compared to all the other passengers, even those who boarded earlier. The unfortunate passengers who board at the end of the line on average end up with almost two inches less spare room compared to the average of the other passengers, and are almost six times more likely to be crowded from the side by the seated girth of seat mates.

We also present here a simple hypothetical, yet unlikely to be adopted, boarding scheme that helps to
optimize the average seat space available to all passengers (no matter their girth) by first sorting them according to seated girth size before boarding, and allowing the largest passengers to board first.

In the following sections we describe the model simulation methods used, and list the modelling assumptions made by this analysis. We then conclude with Results and Discussion.

## Materials and Methods

## Simulation of Seated hip breadth

In 2001, Quigley et al compiled an anthropometric study of human body dimensions, at the request of the EU Joint Aviation Authority commission (Quigley et al., 2001). The study examined the body dimensions of people from various developed countries, including the US. Based on their tabulated information, $1 \% / 5 \% / 95 \% / 99 \%$ of adult Americans have seated hip breadth of at most 12.6/13.5/20.6/23.0 inches, respectively. Under the assumption that the seated hip breadths are Normally distributed, from this data we determine that the average hip breadth is 17 inches, with a standard deviation of 2.16 inches. The resulting distribution of seated hip breadth (girth) is shown in Figure 1, with mean $\mu_{\text {girth }}=17$ inches and standard deviation $\sigma_{\text {girth }}=2.16$ inches. Shown in red are the passengers who do not fit comfortably into airline seats, assuming that the total width available to the passenger is 17 " plus a 2 " armrest.

## Assumptions made in simulation of seating space

In our simulation of free-choice passenger boarding, we simulate seating in airplanes with a $3+3$ seat configuration, which describes the majority of commercial airplanes flying in the US. We assume that the airplane has 24 rows of $3+3$ seats, which describes the configuration of a Boeing 737-300 and 737-700 (we ignore the missing seat in the exit row). Our methods are easily generalizable to different configurations.

We note that aisle and window seats are favored by customers (so much so, that some airlines now charge extra for such seats (Hume, 2012)), likely in part because they afford extra room on the side if the passenger wishes to lean over the armrest. We assume that people sitting in those seats can only be crowded on the side by the seated girth of someone sitting in the middle seat.

We also assume that:

- the plane starts out empty, and is then filled to capacity, with all passengers having only one seat
each,
- all the seats on the plane look alike in terms of size,
- all seats on the plane are of width 17 " plus 2 " armrest,
- all people are traveling alone (i.e. have no a priori preference to sit next to one and other), and are all adults,
- the primary priority a person has in choosing a seat is first to have an aisle or window seat, and next to have a spacious seat (even if it is in the middle),
- and that people are infinitely compressible from the side. If a seat-mate is large enough to invade the seat space of the passenger beside them and "squeeze" them from the side, we equate this with "negative spare room" for both of the passengers. The more "negative spare room", the more forcibly the passengers are squeezed together.

A diagram of the formulas used to compute the available spare space for each passenger in a row of three seats is shown in Figure 2. For all seating arrangements, we simulate 10,000 flights.

## Simulation of Seating: Pre-Assigned Seating

Simulating the distribution of passengers on an airline with pre-assigned seating is trivial; we simply randomly sample the seated hip breadth of each passenger from the Normal distribution with $\mu_{\text {girth }}=17$ and $\sigma_{\text {girth }}=2.16$ inches, then randomly disperse the passengers through the plane.

## Simulation of Seating: Free-Choice Seating

To simulate passengers boarding a free-choice seating airline, we begin by randomly sampling the seated hip breadth of each of the 144 passengers from the Normal distribution with $\mu_{\text {girth }}=17$ and $\sigma_{\text {girth }}=2.16$ inches. We board $1 / 3$ of the passengers first (upon which they fill an aisle seat). The next $1 / 3$ are then allowed to board and choose the window seat in the row that has the most spare room available to them. The last $1 / 3$ then board one by one (there are only middle seats left by this point), and each selects the seat on the airline that provides the most space for them.

## Simulation of Seating: Optimized Seating

We explore the potential of a seating plan that attempts to attain roughly the same amount of room for each passenger. To do this, we first sort the passengers by girth from largest to smallest. The expected value of the $i^{\text {th }}$ out of $n$ ranked Normal random variables (ranked largest to smallest) with mean $\mu$ and standard deviation $\sigma$ is approximately (Royston, 1982)

$$
\begin{equation*}
E(i \mid \mu, \sigma, n) \sim \mu+\sigma \Phi^{-1}\left(\frac{n-i+1-\alpha}{n-2 \alpha+1}\right) \tag{1}
\end{equation*}
$$

with $\alpha=0.375$ (the approximation is best for values of $i$ not close to 1 or $n$ ). We note that

$$
\begin{equation*}
E\left(i \mid \mu_{\mathrm{girth}}, \sigma_{\text {girth }}, n\right)+E\left(n-i \mid \mu_{\mathrm{girth}}, \sigma_{\text {girth }}, n\right)=2 \mu_{\mathrm{girth}} . \tag{2}
\end{equation*}
$$

We also note that the girths of the middle $33 \%$ percentile of passengers sorted by girth are all approximately the same (see Figure 3). Thus if we board the passengers in threes, pairing the largest passenger waiting to board with the smallest passenger waiting to board, along with one of the middle $1 / 3$ of passengers in girth, the amount of room taken up by the three passengers on the seats will always be approximately the same, and equal to $3 \mu_{\text {girth }}$.

To try to achieve this arrangement in a boarding scheme where passengers board one at a time, and also have free-choice in where they sit to increase the speed of boarding, we examine two schemes; the first where passengers are sorted in girth, and allowed to board smallest to largest, and the second where they board largest to smallest.


Figure 1. Distribution of seated hip-width of adult Americans, based on data from Reference (Quigley et al., 2001). Shown in red are the $18 \%$ of people whose girth exceeds the standard 17 inch airline seat width plus 2 inch armrest width.


Figure 2. Diagram of the space available to people sitting in the aisle, middle, and window seats. The girths of the people in the aisle, middle, and window seats are girth $_{\text {aisle }}$, girth $h_{\text {middle }}$, and girth ${ }_{\text {window }}$, respectively, and the seat width is $s_{\text {width }}$. We assume that bodies of larger people can extend, if necessary, past the edge of aisle and window seats with no compression on that side.


Figure 3. Passenger seating scheme that ensures approximately equal spare seating room for all passengers. The smallest passenger sits in same row as largest, along with one of the people from the middle $1 / 3$ of the girth distribution. Then the next smallest and largest passengers are seated with someone else from the middle $1 / 3$ of the girth distribution, and so on.

## Results and Discussion

The average amount of spare room for passengers in the aisle/window and middle seats for the different seating paradigms are tabulated in Table 1. The distribution of room available to the passengers vs boarding position, and the probability of being squeezed vs boarding position, for the various seating paradigms is shown in Figures 4 and 5. The average number of passengers who end up being squeezed per flight is $30 \pm 7,30 \pm 7,15 \pm 5$, and $0.01 \pm 0.09$, out of 144 for the pre-assigned, free-choice, smallest-board-first, and largest-board-first seating paradigms, respectively (the numbers after the " $\pm$ " represent the plus/minus one standard deviation uncertainty). In the following subsections we analyze the results for each the seating paradigms in detail.

Table 1. Summary of average passenger spare room, in inches, for different boarding paradigms on a $3+3144$ seat aircraft.

|  | Aisle/Window <br> avg | Middle <br> avg | First middle <br> seat to board | Last middle <br> seat to board | All seats <br> avg |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Pre-assigned seating | 2.1 | 2.0 | 2.0 | 2.0 | 2.1 |
| Free-choice seating | 2.1 | 2.0 | 3.7 | 0.3 | 2.1 |
| Smallest passengers board first | 2.3 | 1.4 | 3.2 | -0.9 | 2.0 |
| Largest passengers board first | 1.9 | 2.6 | 2.0 | 3.7 | 2.1 |

## Pre-assigned seating

The marginal probability distributions for the spare room of passengers in the aisle/window and middle seats for the pre-assigned seating paradigm is shown in Figure 6. In Figure 6 and Table 1 we note that, on average, people in aisle and window seats have slightly more room than people in the window seats. Under the assumptions that the passenger seat girths really are Normally distributed with mean of $\mu_{\text {girth }}=17$ inches and standard deviation $\sigma_{\text {girth }}=2.16$ inches, we find that the probability of having negative space (being squeezed from the side) is $7 \%$ both for the aisle/window and middle seat passengers. For the middle passengers the average amount of spare room is Normally distributed with $\mu_{\text {spare }}=s_{\text {width }}-\mu_{\text {girth }}$ with standard deviation $\sigma_{\text {spare }}=\sqrt{\frac{3}{8}} \sigma_{\text {girth }}$.

## Free-choice seating

We note in Figure 6 that the marginal probability distribution of spare room for all aisle/window, and all middle seats is exactly the same as that obtained with pre-assigned seating. However, Figure 4 shows
that this perception is somewhat misleading; it is true the distribution is the same for all the aisle/window seats, but people who first begin filling the middle seats have a distinct advantage over those who are last to board, in that they can choose rows where one or more of their neighboring seat occupants are small (thus maximizing their available room). By the time the last people board, all of these favorable configurations have been taken.

The marginal probability distribution of the net amount of spare room available to the passengers in the aisle and window seats in this seating scheme is the same as that of such passengers in pre-assigned seating because the person who chooses the middle seat will always have a girth randomly drawn from the girth probability distribution; thus the probability that the aisle and window passengers will be crowded from the side by the seated girth of the middle seat passenger is exactly the same as it would be for pre-assigned boarding. The marginal probability distribution for the net amount of spare room for the passengers in the middle seat is also the same as that of such passengers in pre-assigned seating because even if the middle seats are ranked by available room from from most to least, the girth of passenger who chooses a particular seat is randomly drawn from the girth probability distribution, just as it is in pre-assigned seating.

However, the first middle seat passenger to board will be able to choose the middle seat with the most room, while the last passenger to board will find all of the roomiest middle seats taken. This leads to disparities in the amount of net spare room that each of the middle passengers get; once all the aisle and window passengers have boarded the probability distribution for the available seating room between them (without a middle passenger in the seat) is Normally distributed with mean

$$
\begin{equation*}
\mu_{\text {middle }}=2 s_{\text {width }}-\mu_{\mathrm{girth}} \tag{3}
\end{equation*}
$$

and standard deviation

$$
\begin{equation*}
\sigma_{\text {middle }}=\sigma_{\text {girth }} / \sqrt{(2)} \tag{4}
\end{equation*}
$$

We calculate the expected value of the available room in the $i^{\text {th }}$ ranked middle seat (from largest to smallest in available room) using Equation 1. The net amount of spare room for the $i$ middle seat
passenger once they are seated is thus Normally distributed, with mean

$$
\begin{equation*}
\mu_{\text {spare }}\left(i \mid \mu_{\text {middle }}, \sigma_{\text {middle }}, n\right)=\frac{E\left(i \mid \mu_{\text {middle }}, \sigma_{\text {middle }}, n\right)-\mu_{\text {girth }}}{2}, \tag{5}
\end{equation*}
$$

and standard deviation

$$
\begin{equation*}
\sigma_{\text {spare }}=\sigma_{\text {girth }} / 2, \tag{6}
\end{equation*}
$$

where $E\left(i \mid \mu_{\text {middle }}, \sigma_{\text {middle }}, n\right)$ is calculated with Equation 1. The prediction for the net amount of spare room from Equation 5 is shown in Figure 4, overlaid over the distribution of net amount of spare room vs boarding position of the middle seat passengers in the agent based boarding simulation.

Thus, we find that the choicest boarding position is not at the beginning of the line, because those people have no control over who decides to sit next to them, and thus fare no better than people in an assigned-seating airline. The best boarding position in free-choice seating is $2 / 3$ down the line, once all aisle/window seats have been filled, and the passenger has the choice of the most spacious middle seat.

## Optimized seating schemes

As we have just seen, in a free-choice boarding scheme, the seating comfort of a few is gained potentially at the expense of others. Here we examine the potential of two boarding schemes that involve sorting the passengers by size before they board the plane.

The distribution of spare room vs boarding position for passengers in the aisle/window and middle seats is shown in Figure 6. The results are tabulated in Table 1. The marginal distribution of the spare room for both seating schemes, as compared to the pre-assigned and free-choice seating schemes, are shown in Figures 7 and 8.

## Simulation of 10000 flights



Figure 4. Amount of spare room, in inches, vs boarding position for various boarding schemes. We assume that the aisle and window seats are filled first, followed by the middle seats. A total of 10000 flights of a $3+3144$ seat aircraft are simulated. Overlaid in green are the expected values obtained using average rank Normal values calculated using Equation 1.

Probability of Being Squeezed vs Boarding Position


Figure 5. Probability of being squeezed from the side, vs boarding position for various boarding schemes.


Figure 6. Probability distributions of the spare room available to passengers in the aisle/window and middle seats for pre-assigned seating and free-choice boarding schemes.


Figure 7. Marginal probability distributions of the spare room available to passengers in the aisle/window and middle seats for pre-assigned seating and smallest-people-board-first boarding schemes.


Figure 8. Marginal probability distributions of the spare room available to passengers in the aisle/window and middle seats for pre-assigned seating and largest-people-board-first boarding schemes.

## Summary

In this analysis, we examined two common aircraft boarding paradigms (pre-assigned seating, and freechoice seating) and found that at first glance they appear to provide the same amount of space to passengers. However, the space available to people on a free-choice boarding flight strongly depends on their boarding order. The first $2 / 3$ of people who board have no advantage over one and other, and no space advantage over people who fly traditional assigned seating airlines. Once the first $2 / 3$ of the people have boarded and all the aisle and window seats are filled up, the very next person to board has the opportunity to choose the most spacious middle seat available. There is thus a strong incentive to obtain a boarding position $2 / 3$ of the way down the line. However, to the author's knowledge, the only boarding position upgrades that free-choice boarding airlines such as Southwest offer is to the first positions in the line. Free-choice boarding is disadvantagous to those who fall in the last $1 / 6$ of the boarding lineup; these people are very likely to have negative spare room and poor comfort during the flight.

Our results are based on simulations that are somewhat simplistic in that they assume that passenger seating preferences are primarily driven by a desire to optimize spare seating room, and that passengers do not sit in pre-arranged groups (ie; people traveling together).

We proposed a novel boarding method, whereby the passengers were sorted according to size, and the largest board first. We showed that this paradigm resulted in a low probability of any passenger not having enough room. However, the paradigm suffers from implementation issues; many would likely find it degrading to be sorted by size, and smaller people may be displeased to consistently board last (even though it ensures them more space in the end).

While social norms make public acceptance of such a scheme extremely unlikely, despite its advantages, our studies underline the utility that mathematical and computational modelling can have in developing optimal solutions for the comfort and well-being of air travelers.

## Key points:

- We used an agent-based computational model to compare passenger seating comfort (based on the probability of being crowded from the side by the seated girth of another passenger) in free-choice and pre-assigned seating paradigms (FCSP and PASP).
- Passengers in the first $2 / 3$ of the boarding lineup in FCSP fare no better in seating comfort than aisle and window passengers in PASP, and
- The boarding position that provides optimal passenger seated room in FCSP is just past $2 / 3$ down the boarding lineup.
- Overall seating comfort is optimized in FCSP when passengers are sorted by size, and large passengers board first.


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